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Abstract

A major problem in studying the performance of private equity is the lack of reliable market data, or the lack of liquidity. In addition, it is very difficult to clearly evaluate the performance of single private equity investments since no benchmark based on market prices has been built so far for this asset class. This paper addresses these issues by developing different indices for publicly traded private equity (PTPE) vehicles. We find 287 companies being listed in the time period from 1986 to 2003. After imposing liquidity constraints with respect to availability of market prices, capitalization, trading volume, bid-ask spread, and trade-continuity, and after correcting for non-surviving vehicles, we get a sample of 114 instruments. The risk and return characteristics of two partially rebalanced indices (one equally weighted, the other value weighted) and of a fully rebalanced equally weighted index are investigated. Special attention is given to potential biases with respect to thin trading, the bid-ask spread effect in rebalancing, and sample selection. Adjusted performance estimates differ substantially from standard estimates of risk and return. Even after correcting for these biases, we find a high risk-adjusted performance of this asset class before 2000, and dramatic different results between the three indices if we extend the time period to 2003.

Keywords: Publicly traded private equity, Private equity, Benchmark index, Performance biases

JEL classification code: G24

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1. Introduction

Private Equity has become a very popular research topic in the past years. However, performance characteristics of this asset class still have not been fully understood yet. One reason for this is that the empirical results of almost all studies are based on book values, not market values. A different approach is taken in this paper. We analyze the risk and return of *publicly traded* private equity vehicles which constitute an attractive and rapidly growing segment within the overall private equity market. Obviously, the advantage of this market segment is the availability of market prices which enables more reliable performance measures. Since private equity is by definition a business area that deals with investments in non-listed companies, we were surprised to identify a total of 287 private equity vehicles listed on worldwide stock exchanges. We were thus able to construct a representative index which can be used to study the risk and return characteristics of this asset class, and which serves as a valid benchmark for comparisons with traditional investments.

There are only a few academic studies on the risk and return characteristics of publicly traded private equity companies, and they date back to the 1980s. Martin/Petty (1983) identify 37 venture capital firms during 1970 and 1980 but only 17 of them have price data available from 1974-1979. Another 6 companies are not analyzed due to inactive trading. A liquidity constraint is implemented: a firm has to have at least 60% of its monthly returns evidencing a price change. At the end their dataset consists of only eleven companies but no portfolio is formed out of the venture capital firms. Brophy/Guthner (1988) provide estimates of risk and return and compute beta relative to the S&P500 index for twelve funds using weekly data from 1981 to 1985. They also calculate returns of different portfolios of these companies coming up with the result that these portfolios achieve superior returns compared to the S&P500 and to growth-oriented mutual funds.

But none of these studies investigates the risk and return characteristics or the investment behavior of PTPE for a larger sample size and for a longer time period. Also the impact of the down market we have seen from 2000 until today has not been part of any study. However, there are several papers that concentrate on the risk and return characteristics of *non-quoted* private equity. Bygrave and Timmons (1992) study the performance of venture capital funds for 1974-

1989. They calculate an internal rate of return of these funds of 13.5% without providing any risk calculations. Another study that measures risk and return in the context of private equity is the one of Gompers and Lerner (1997a). Their findings of an arithmetic average annual return of 30.5% and their beta estimations are based on the examination of the investments of only one venture capital firm. Long (1999) studies nine VC investments and comes up with a standard deviation of 8.23% per year.

Moskowitz/Jorgensen (2000) find that the returns to private equity are much lower than one would expect. The average annual return to all privately held companies is about two to three percentage points below the return of an index of publicly traded equity over the period 1989 to 1998, and is similar to the public market over the period 1964 to 1998. An index of private equity is also more volatile than a public equity index and is also highly correlated with public market returns. Hence they ask why an investor would invest large amounts in a single privately held firm with a far worse risk-return trade-off. They conclude that only non-pecuniary benefits and/or overestimation of the entrepreneurs' success can explain the concentration of wealth in private equity.

Cochrane (2004) measures the mean, standard deviation, alpha and beta of venture capital investments after correction for selection bias[6]. The results are based on the VentureOne database from 1987 to 2000. His results are as follows: without a selection bias correction he finds a mean log return of 108%. With the bias correction he finds a mean log return of about 15%. The arithmetic average return is 698% without a correction for selection bias and 59% with a correction for the bias. Peng (2001) who uses the same database as Cochrane (2003) reports a geometric average return from 1987 to 1999 of 55.18%. This is much higher than the corresponding result of Cochrane. However Peng does not correct for selection bias.

Ljungqvist and Richardson (2003) provide the first analysis of private equity returns based on actual cash flows of venture capital and buyout funds. They find that private equity has generated substantial excess returns over the past two decades, find an internal rate of return of 19.8% and

[6] This kind of selection bias leading to an upward bias in performance estimates arises when only instruments are studied which have been listed due to high performance.

state that this excess return compensates the investor for holding an illiquid investment for a certain time period.

In contrast to those studies shortly presented above this paper will focus on listed private equity vehicles. In Section 2, we provide a detailed description of our database which covers the most complete sample of PTPE vehicles analyzed in the literature so far. This Section also contains our definition and selection criteria for PTPE which is consistently used throughout the sample period from 1986 to 2003. At the end of this Section, we define and evaluate several minimum liquidity conditions that reduce the basic sample of 287 vehicles to a restricted portfolio of 122 liquid instruments. In Section 3 we investigate basic risk and return characteristics of three different portfolio strategies (indices) applied to the restricted sample. In Section 4 we discuss three potential biases in computed risk and return figures, caused by thin trading, the bid-ask spread, and sample selection/survivorship. In Section 5 we briefly address problems related to the creation of a benchmark index for the PTPE asset class. Section 6 concludes the paper.

2. Database and sample characteristics

2.1 The Basic Sample

In this paper, instruments are classified as PTPE if the underlying business is PE investing, but the funds themselves are quoted on an exchange. Companies which only partly invest in private equity, e.g. investment banks, holding companies, venture capital pools[7] and the likes were excluded. The respective instruments fall into three categories:

- listed companies whose core business is private equity (e.g. 3i),
- quoted investment funds, which invest a predetermined proportional equity share to specific private firms together with the company's private funds (i.e. Schroder Ventures Trust),
- or specially structured investment vehicles which invest in private equity directly (e.g. investing into private companies), and/or indirectly through various private funds. (i.e. Castle Private Equity).

[7] A special structure of the CDNX.

The underlying private equity investments of these instruments include all kinds of possible financing stages and styles. Portfolio companies of these vehicles receive financing in the early, later and expansion stage, but also buyouts and turnaround situations are financed.
Based on this definition, we find a total of 287 listed instruments between 1986 and 2003. At the beginning of the sample period, only eight instruments existed, which shows the dramatic growth in this market segment. From the 287 instruments, only 237 satisfy our definition over the whole observation period. The remaining 50 vehicles were either acquired, delisted from the exchange, or changed their businesses, and must therefore be studied separately. Simply neglecting them would lead to a serious selection bias.

Some structural characteristics of the sample are displayed in **Exhibit 2**. Most vehicles (i.e. 173 of the 287) have been listed over the past six years only, many of which concentrating on the financing of young technology firms. In contrast, only five were listed in 1973. In the late eighties, there was a wave of listings (35), mainly in Europe, concentrating on management buyouts. Over 50% of the vehicles are listed in Europe, most of them in the UK (113) where Investment Trusts and Venture Capital Trusts are widely spread because of tax alleviation. Almost 30% are listed in North America and more than 10% in Asia. In terms of (average) market capitalization 38.23% of the overall market value is quoted in Europe and 24.90% in North America. The median of only $22 million shows that most companies have a rather small market capitalization. **Appendix 1** shows the details of the country classification.

As with other alternative investments, a major practical restriction of PTPE investments is illiquidity. Requiring specific minimum liquidity conditions is important for comparing the risk and return measures of PTPE with traditional stock market investments. However, unlike other alternative investments, our asset category are listed instruments, and liquidity can be measured by various *market* characteristics. Liquidity has many different faces – but for our purposes, it is sufficient to capture the multidimensional character of liquidity by investigating a series of market ratios. Our selection process is based, somehow arbitrarily, on the following criteria:

1. We require complete series of weekly price observations in order to ensure accuracy of parameter estimates.
2. The vehicles must have a minimum average market capitalization.

3. To assure a minimal trading activity, we impose a restriction on the average of the relative trading volume per week t which is defined by

   $$\text{Relative volume}_t = \frac{TradingVolume_t \, x \, \text{Price}_t}{MarketValue_t}$$

4. A certain trade continuity has to be assured. This is measured by the percentage of weeks within which at least one transaction occurs.
5. Finally, we require a maximum average bid-ask spread, which we define as

   $$\text{Spread}_t = \frac{(Ask_t - Bid_t)}{Mid \, \text{Price}_t}$$

Before applying the various liquidity constraints to our 287 instruments, we look whether they in fact capture different sample characteristics. **Appendix 3** shows that the correlation coefficients between the criteria is relatively low, except for the bid-ask spread which has a correlation of -0.48 vis-à-vis the "market capitalization" parameter.

2.2 The Restricted Sample

Imposing minimum liquidity constraints narrows our sample considerably. This creates, of course, a trade-off between the sample size and the liquidity of the vehicles in the resulting sample. We have to try out different parameter specifications to see how the size of the resulting sample is affected (**Appendix 4**). We finally impose the following, moderate constraints:

- a minimum of 30 weekly observations
- an average market value of minimum $2 million
- a minimum relative trading volume of 0.1% per week[8]
- an average bid-ask spread smaller than 20%
- a minimum continuity of trade of 15%[9]

This results in a total of 122 instruments in the sample, i.e. 165 or 57% of the vehicles are excluded from the original sample due to the previous constraints. It is apparent that illiquidity is a serious issue even in the segment of traded private equity investments. Notice that of these 122

[8] Compared to the relative trading volume of 2.1% per week of AIM companies in the first 6 month of 1996 which was analyzed by Board/Vila/Wells (1998).
[9] Board/Vila/Wells (1998) find that in the first 6 months of 1996 AIM companies were traded on 52% of all days, on average.

companies, eight have been liquidated, changed their business or have been acquired by another company. Therefore, the empirical analysis of Sections 3 and 4 will be based on 114 companies.

Exhibit 1: Liquidity constraints

Number of vehicles	Liquidity constraints				
	Minimum of weekly observations	**Minimum average market capitalization**	**Minimum relative trading volume**	**Minimum continuity of trade**	**Maximum average bid-ask spread**
122	30	2	0.1%	15%	20%

Exhibit 2: Distribution and characteristics of Publicly Traded Private Equity Companies, 1986-2003

year of listing	number of companies	Percentage
prior to 1986	13	4.53%
1986	3	1.05%
1987/1988	17	5.92%
1989/1990	18	6.27%
1991/1992	6	2.09%
1993/1994	15	5.23%
1995/1996	42	14.63%
1997/1998	54	18.82%
1999/2000	98	34.15%
2001/2002	21	7.32%
Total	**287**	**100.00%**

Region	number of companies	percentage	market value [Mio. US$]	percentage
America	82	28.57%	13'645.84	24.90%
Asia	31	10.80%	19'741.37	36.02%
Europe	163	56.79%	20'951.15	38.23%
Rest of the world	11	3.83%	466.86	0.85%
Total	**287**	**100.00%**	**54'805.22**	**100.00%**

Datasource: Primark Datastream

3. The risk and return of PTPE

In this Section, we investigate the risk and return characteristics of portfolios, or indices, constructed from the 114 vehicles included in our liquid sample. There are several sample characteristics to be considered in this context: First, as shown in Section 2, listings occur steadily over time. As mentioned earlier, at the beginning our observation period, only eight instruments were available. In order to analyze the performance of all vehicles over the whole observation period, the portfolio must be *rebalanced* whenever new listings take place. Second, the sample is extremely heterogeneous with respect to *market capitalization* of the individual firms. Also, the evolution of capitalization over time (i.e. the performance of the companies) is very heterogeneous, including the whole range from high-fliers to flops. Third, there is the challenge of taking the limited liquidity of PTPE vehicles into account. Although we have imposed minimum liquidity constraints, we must still be aware that our sample of firms is less liquid than "traditional" stocks. This fact substantially affects the measurement of performance, and we will address this issue separately in Section 4 of this paper.

3.1 Three indices

Performance measurement of asset classes is mostly based on indices, which can be either value[10]-, price[11]-, or equally weighted. Moreover, the indices can be either (re-)balanced or unbalanced (buy and hold), or a mixture thereof. The type of index reflects the underlying investment style. While the weighting scheme is not always important when analyzing indices of traditional stocks, the index choice extremely matters in our context given the sample characteristics mentioned before. We investigate three types of indices in order to capture the different characteristics adequately:

- *A value weighted PTPE index, buy-and-hold (partially rebalanced): VW-BH*
  The weights of this index are determined by the relative market capitalization of the individual instruments. The value of the index at time t is computed by

[10] E.g. the S&P 500.
[11] E.g. the Dow Jones Industrial Average.

(1) $$I_t = I_{t-1} \times \sum_{i=1}^{n} \frac{M_{it}}{M_t} \times \frac{(P_{it} + D_{it}) \times Adj_{it}}{P_{it-1}}$$

where I_t is the value of the Index at time *t*, M_{it} is the market value of vehicle *i* at time *t*, M_t is the sum of the market values of all vehicles at time *t*. P_{it} and D_{it} are the price and the dividend of the vehicle *i* at time *t*. Adj_{it} is an adjustment factor in case of stock split or capital increase. The index basically represents an unbalanced strategy, except if a new listing occurs: capital is taken out of the existing vehicles and reinvested in the new portfolio constituent. A characteristic of the index is that market capitalization varies considerably over time, and across different vehicles. The index naturally allocates excessive weight to only a few companies, which is regarded as a disadvantage by many investors.

- *An equally weighted PTPE index, rebalanced: EW-RB*

  An equal fraction of wealth is allocated to the individual instruments. In order to maintain constant equal weighting over time, the index is rebalanced on a weekly basis. The index is computed by

(2) $$I_t = I_{t-1} \times \frac{1}{n} \times \sum_{i=1}^{n} \frac{(P_{it} + D_{it}) \times Adj_{it}}{P_{it-1}}$$

where I_t is the value of the index starting at time t, n is the number of vehicles listed at time t, P_{it} and D_{it} are the price and the dividend of the vehicle i at time t and Adj_{it} is an adjustment factor in case of stock split or capital increase.

- *An equally weighted PTPE index, buy-and-hold (partially rebalanced): EW-BH*

  In order to compare the actively rebalanced, EW-RB index with a similar, but more passive strategy we construct an index representing a buy and hold strategy. The index constituents are equally weighted at the beginning of our observation period (1986), but there is no weekly rebalancing. The only rebalancing to equal weights occurs when new vehicles are listed.

Exhibit 5 highlights the evolution of the three indices from 1986 to 2003. Risk and return statistics such as means, standard deviations, Sharpe ratios, alphas, betas, and autocorrelation coefficients are displayed in **Exhibit 4**. All figures refer to continuously compounded rates of return.

3.2 Risk and return characteristics

It is apparent that the volatility of the (partially rebalanced) VW-BH index is more than twice as large as for the (fully rebalanced) EW-RB index, and is still substantially larger than the (partially rebalanced) EW-BH index. This can be explained by a lack of diversification of the value weighted index. In fact, 79.39% of the average market capitalization of the 114 vehicles in the VW-BH index is represented by only ten vehicles, while the 72 smallest instruments account for only 5% of the market cap. The diversification effect of the rebalancing strategy is also reflected in a higher beta of the VW-BH index (computed with respect to the MSCI (World) index). It is thus questionable whether the VW-BH index is regarded as a valuable benchmark by investors seeking diversification in this asset class.

Over the whole observation period from 1986 to 2003, the average annual return is clearly highest for the (fully rebalanced) EW-RB index (15.99%), compared to the values of the buy-and-hold indices: 5.43% for the VW-BH index, and 5.91% for the EW-BH index. These figures show the dramatic impact of rebalancing in this market segment. Splitting the sample in two subperiods (the first representing the boom market up to the year 2000, the second representing the down market after March 2000), reveals the dramatic downturn of the private equity market, but again, confirms the substantial benefit from rebalancing. The loss represented by the weekly rebalanced EW-RB index (-35.39%) is substantially smaller than for the buy-and hold indices: -58.39% for the EW-BH index, and –68.93% for the VW-BH index.[12]

Obviously, the risk-adjusted performance measures confirm these findings. The Sharpe ratio of the EW-RB index is 1.19 for the time period 1986 to 2000 and clearly exceeds the ratio found for the MSCI (World) index (0.47). The Sharpe ratios of the other indices are virtually zero. Jensen's

[12] See e.g. Bernstein/ Wilkinson (1997) for a more detailed characterization of the rebalancing bonus.

alpha for the time period 1986 to 2003 is 10.18% for the EW-RB index, essentially zero for the EW-BH index, and –1.20% for the VW-BH index. The MSCI (World) index is used as the benchmark portfolio. The R^2-coefficients of the underlying regression are rather small (see **Appendix 6**). This shows that private equity exhibits a large, specific and thus, diversifiable variance component with respect to standard equity investments. This is also reflected in the beta coefficients, which are, for the EW-RB and EW-BH index returns, significantly below one (0.60 and 0.70, respectively). One is tempted to conclude that PTPE is not exposed to so much risk as generally assumed.[13]

Exhibit 4 finally displays the autocorrelation structure of the index returns. Interestingly, the EW-RB index exhibits statistically significant positive serial correlations up to five lags, which is the consequence of a thin trading effect due to relative large weight of small firms in equally weighted indices. Consistent with this interpretation, the value weighted index (VW-BH) exhibits much less serial correlation. The bias induced by serial correlation for estimating return volatility will be discussed in Section 4.

Finally, it is not surprising that the results of the equally weighted and value-weighted (EW-BH and VW-BH) indices are rather similar. This is due to the fact that over time, the weighting of the buy-and-hold index converges to the value-weighted equity mix.[14] Therefore, some of the shortcomings of the VW-BH index also apply to the equally weighted counterpart. This is especially true in the period after 2000, when only a few new listings are observed.

[13] Risk of private equity investments is often stated as rather high: See French (1988). Yet, measuring risk with the volatility of private equity is very problematic when estimated from private investments because accounting data does not reveal the relevant risk exposure. Therefore, volatility of private equity is usually underestimated in *the generally understood* sense. Estimations of risk from publicly traded equity give us a better indicator for the real risk exposure. However, volatility from observed market prices still does not account for the whole risk exposure. Compared to the highly liquid stock market indices our PTPE index is rather illiquid. We will discuss this issue in section 4.

[14] See Roll (1983).

Exhibit 4: Risk/return analysis of publicly traded private equity (PTPE), 1986-2003

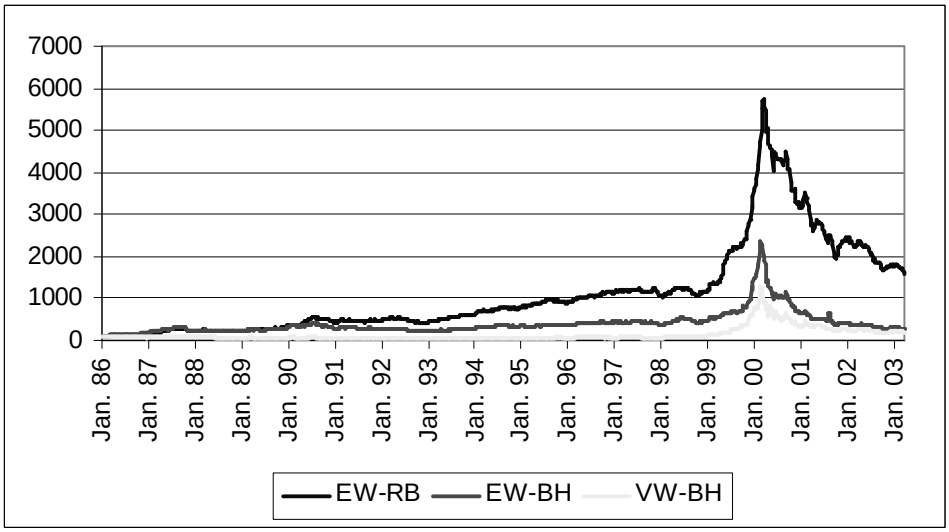
value weighted buy-and-hold (VW-BH)		US$ Mean	SD	Sharpe Ratio	Alpha	Beta	Autocorrelations AC1	AC2	AC3	AC4	AC5
	1986 - 2003	*5.43%*	*43.18%*	*0.01*	*-1.20%*	*1.20*	*0.114***	*-0.012*	*0.053*	*0.037*	*0.075**
	1986 - 2000	19.37%	40.77%	0.35	7.16%	1.08	0.140**	0.023	0.070	0.145**	0.010
time period	2000 - 2003	-58.39%	52.07%	n.c.	-31.18%	1.41	0.028	-0.130	-0.001	-0.220**	0.273**

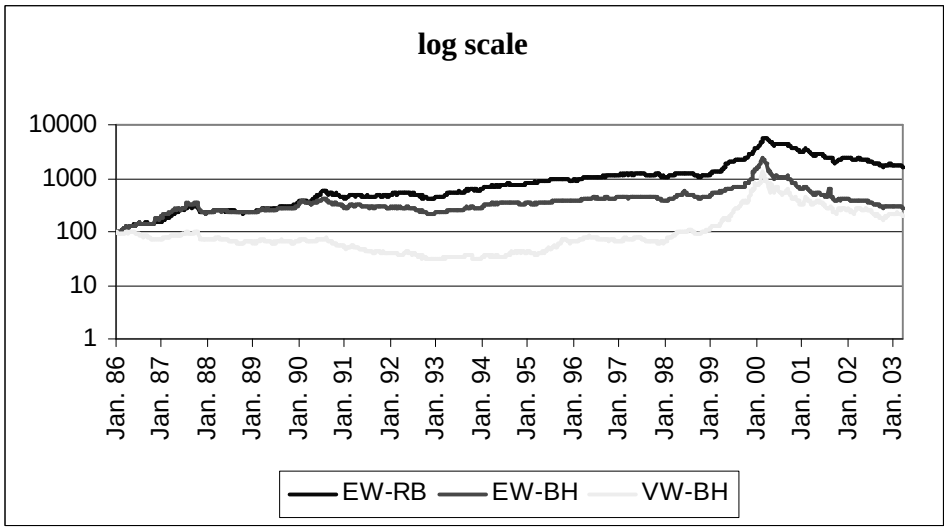
equally weighted fully rebalanced (EW-RB)		US$ Mean	SD	Sharpe Ratio	Alpha	Beta	Autocorrelations AC1	AC2	AC3	AC4	AC5
	1986 - 2003	*15.99%*	*19.34%*	*0.57*	*10.18%*	*0.60*	*0.151***	*0.159***	*0.115***	*0.099***	*0.077**
	1986 - 2000	27.21%	18.44%	1.19	18.21%	0.58	0.107**	0.123**	0.123**	0.104**	0.088*
time period	2000 - 2003	-35.39%	21.68%	n.c.	-25.83%	0.60	0.223**	0.159*	-0.049	-0.056	-0.092

equally weighted buy-and-hold (EW-BH)		US$ Mean	SD	Sharpe Ratio	Alpha	Beta	Autocorrelations AC1	AC2	AC3	AC4	AC5
	1986 - 2003	*5.91%*	*26.93%*	*0.04*	*-0.09%*	*0.74*	*0.058*	*0.150***	*0.007*	*0.072**	*0.098***
	1986 - 2000	22.26%	22.81%	0.74	12.37%	0.72	0.051	0.143**	0.098**	0.078*	0.096**
time period	2000 - 2003	-68.93%	39.24%	n.c.	-56.74%	0.72	-0.001	0.119	-0.201**	0.018	0.091

*/** denotes that the correlation is significant at the 5%/1% level.
Annualized weekly returns are computed from 1st January 1986 - 19th March 2003, 1st January 1986 - 16th February 2000 and 16th February 2000 – 19th March 2003. They are based on 114 liquid companies.

Exhibit 5: Development of the PTPE indices (January 1986 to March 2003)*





*An equal amount of capital is invested in each index at the beginning of the observation period. The EW-RB index is weekly rebalanced to equal weights while the other two indices, the EW-BH as well as the VW-BH index are only rebalanced in the case of a new listing. The indices are based on 114 liquid companies.

4. Performance biases – revised measures of risk and return

In the previous Section, we referred to various sample characteristics (high cross-sectional and time-series variability of the market cap, limited liquidity of the index constituents, the need for rebalancing) which have the potential to induce various biases in the risk and return characteristics of PTPE. Concerning the first issue we pointed out that a value weighted index is probably not a well suited benchmark for PTPE since it puts excessive weights to only a few vehicles. The main focus in our following analysis is therefore on equally weighted indices.

In this Section, the potential biases emerging from the following issues are addressed:

1. **Limited liquidity:** Although we have already imposed several liquidity constraints on our original sample, the liquidity of the selected vehicles is still very limited. Getmansky, Lo and Makarov (2003) show that the most likely source of serial correlation in hedge fund returns is due to illiquidity. They also point out that portfolios of illiquid securities tend to be smoother than true economic returns, which understates volatility and increases risk-adjusted performance measures such as Sharpe ratios. As PTPE returns show the same pattern, the potential bias due to positive serial autocorrelation will be analyzed.

2. **Rebalancing:** The need for rebalancing arises if the capital invested in each stock is kept constant over time. The rebalancing strategy implemented here assumes that the index portfolio is weekly rebalanced to equal weights. Accordingly, better performing stocks are sold, and bad performers are bought, respectively. With substantial illiquidity such as high bid-ask spreads, this procedure can be extremely costly.

3. **Selection bias:** In the previous Section, we have studied the performance of the 114 stocks which have survived over the entire sample period. The potential survivorship bias implied by this procedure generally leads to an upward bias in computed returns.[15] A second selection bias could be present in our data because, at the beginning of the sample time period, there was no clear common definition of our asset class in the literature.

[15] See e.g. Elton/Gruber/Blake (1996).

The purpose of this Section is to address these issues in depth, and to come up with adjusted performance measures.

4.1 Volatility bias – correction for autocorrelation

As observed in the previous Section, the EW-RB as well the EW-BH index returns reveal strong autocorrelation in weekly returns. In **Appendix 5** the autocorrelation structure is displayed in more detail, together with the autocorrelation coefficients of the MSCI (World) returns. For the EW-RB index, we find statistically significant positive autocorrelations at the first five lags, and the EW-BH index exhibits significant positive autocorrelations at the second, fourth and fifth lag.[16] The Ljung-Box Q-statistic clearly rejects the null hypothesis of zero autocorrelation for the first five lags. In contrast, there is no significant autocorrelation in the weekly returns of the MSCI (World) index. There is a vast body of literature on autocorrelation patterns in stock returns. For *weekly* returns, Campbell/Lo/MacKinlay (1997) report statistically significant first-order autocorrelation for the CRSP equally-weighted index of 0.203 from July 1962 to December 1994.

Several theoretical explanations are offered in the literature: First, autocorrelation structures are consistent with time-varying risk premiums. Second, autocorrelation is explained by the existence of market frictions caused by microstructure effects such as thin trading, or institutional structures such as differences in trading mechanisms, or by measurement errors in the data such as non-synchronous trading and bid-ask spreads. Examples for explicit models of thin trading (or non-trading) can be found in the studies of Scholes and Williams (1977), Dimson (1979) or Campbell/Lo/MacKinlay (1997). A third group of theoretical explanations states that autocorrelation results from market inefficiency in the sense that some stocks either overreact, or adjust slowly, to new information arriving on the market.[17]

[16] We only measure the *linear* dependence of concurrent returns on lagged returns. For the first and third week, we do not find statistical significance for linear dependence of concurrent returns which does not exclude any non-linear dependence.
[17] See e.g. Lo/MacKinlay (1990).

Observing non-zero autocorrelation in PTPE returns affects risk and performance measures. Positive autocorrelation, in particular, leads to a downward bias in estimated risk parameters. Mathematically, the problem can be stated as follows[18]: The weekly returns $r_{i1,}r_{i2},.....,r_{iT}$ are represented by a sample of size T from a covariance-stationary process with $E(r_{it}) = m$. Define the autocovariances as

$$(3) \qquad E\left((r_{it} - m)(r_{i,t-j} - m)\right) = g(j), \qquad \sum_{j=0}^{\infty} |g(j)| < \infty \text{ for all } t.$$

Setting the mean equal to zero for simplicity (and not much loss of generality, because our observation interval, and thus, the returns are small) implies the following estimate of the annualized variance of returns, adjusted for autocorrelations:

$$(4) \qquad \hat{s}_i^2(r_i) = E\left[\left(\sum_{t=1}^{T} r_{it}\right)^2\right] = T \cdot \sum_{j=-\infty}^{K=\infty} g(j)$$

A problem arises since it is not possible to set K=infinite since the number of observations is limited. French/Schwert/Stambaugh (1987), for example, set K=1. They argue that in the case of non-synchronous trading, daily returns are autocorrelated particularly at lag one. For weekly returns, Lo/McKinlay (1988) (1990) report (small) positive autocorrelation at lags two to twelve, but negative autocorrelation (mean-reversion) for measurement intervals over one quarter. This leads to the difficulty of identifying the appropriate number of lags. Including too many lags could lead to excessive noise in the estimated variances. We somehow arbitrarily choose to set K=20.

Exhibit 6 presents the autocorrelation-adjusted variance estimates, as well as the adjusted Sharpe ratios, betas[19], and alphas, as compared to the standard performance estimates. The adjusted risk estimates are considerably higher for both indices: the standard deviation for the full 1986-2003 period is 33.69% (37.09%) instead of 19.34% (26.93%) for the EW-RB (EW-BH) index. The

[18] See Hamilton (1994), p. 188.
[19] We run the regression of vehicle returns against three lagged, matching and three leading returns. See Dimson (1979).

respective beta coefficient is 0.99 (1.09) instead of 0.57 (0.47). The adjusted Sharpe ratio is 0.33 (0.03) instead of 0.57 (0.04), while the alpha is surprisingly virtually unchanged: 10.43% (-0.05%) instead of 10.18% (-0.09%).

The conclusion is that the adjustment for autocorrelation has a substantial impact on the total and systematic risk estimates of private equity, as well as on the Sharpe ratio. However, compared with the Sharpe ratio of MSCI (World) stock market returns which is 0.09 over the 1986 to 2003 time period, the reward to risk of an EW-RB PTPE portfolio is still by far superior even after correcting for the volatility bias.

A major limitation is, however, that the adjustment only applies to covariance-stationary processes. Without providing a formal test for this property, this is obviously a very critical assumption for our returns. As it is apparent from **Exhibit 4**, the volatilities as well as the autocorrelations differ between the two subperiods. Therefore, our adjusted estimates should be regarded as illustrative, serving to highlight the impact of non-zero autocorrelations induced by thin trading for the case of PTPE.

Exhibit 6: The volatility bias in PTPE: Adjustment for autocorrelation

EW-RB	Standard Estimates US$					Adjusted Estimates US$			
	Mean	SD	Alpha	Beta	Sharpe Ratio	Adjusted SD	Adjusted Sharpe Ratio	Dimson Alpha	Dimson Beta
Time Period 1986 – 2003	15.99%	19.34%	10.18%	0.60	0.57	**33.69%**	**0.33**	**10.43%**	**0.99**

EW-BH	Standard Estimates US$					Adjusted Estimates US$			
	Mean	SD	Alpha	Beta	Sharpe Ratio	Adjusted SD	Adjusted Shape Ratio	Dimson Alpha	Dimson Beta
Time Period 1986 – 2003	5.91%	26.93%	-0.09%	0.74	0.04	**37.09%**	**0.03**	**-0.05%**	**1.09**

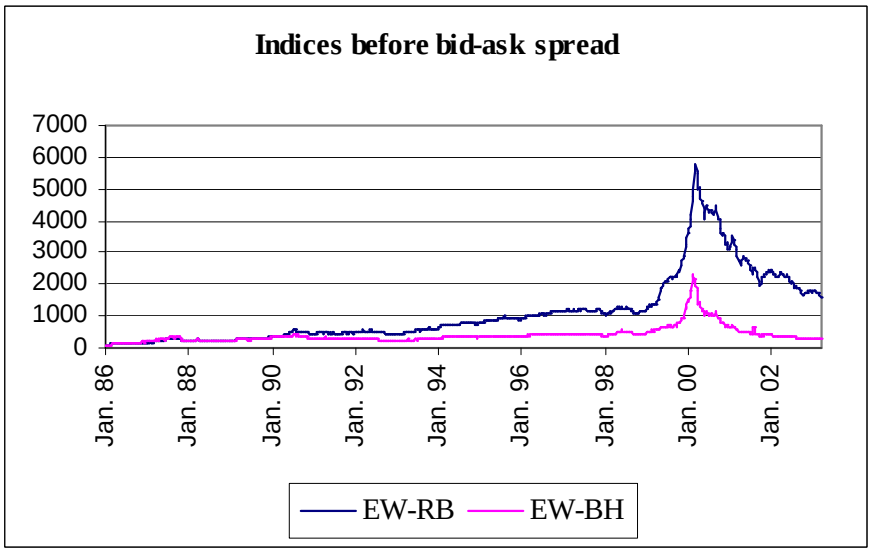
Annualized weekly returns are computed from 1st January 1986 - 19th March 2003, 1st January 1986 - 16th February 2000 and 16th February 2000 – 19th March 2003. They are based on 114 liquid companies.

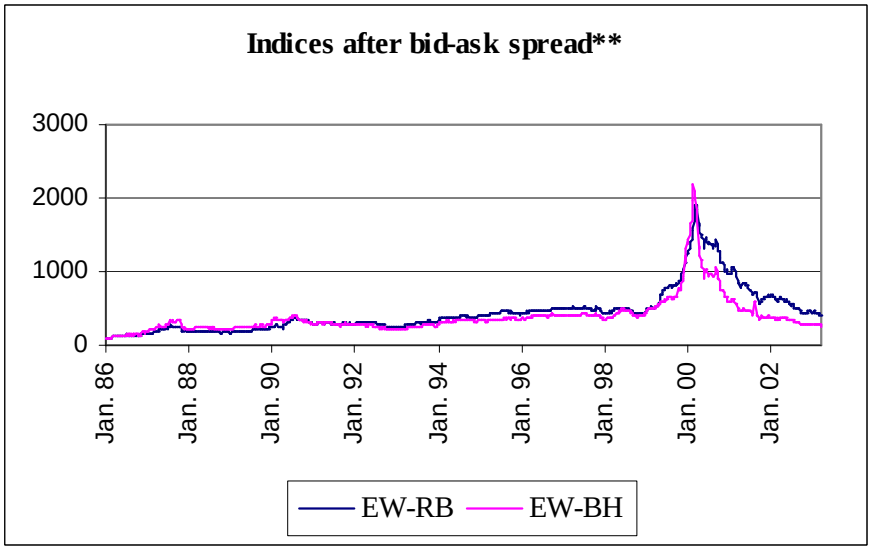
4.2 The bid-ask bias – the rebalancing issue

The motivation for rebalancing arises from keeping the relative fraction of wealth invested in each vehicle constant over time. Remember that 2 rebalancing strategies are examined in this paper: The EW-RB index is weekly rebalanced to equal weights, and the EW-BH index is only rebalanced if a new listing occurs. We have shown in **Exhibits 4 and 6** that the EW-RB index not only shows a better risk adjusted performance, but also considerably higher average returns. There seems to be a "rebalancing bonus", which has already been discussed in the literature. Early papers of Blume and Stambaugh (1983) and Roll (1983) discuss the problem of estimating and comparing rebalanced and buy-and-hold mean holding period returns. Blume and Stambaugh explain the difference between these mean returns by the use of closing prices, which differ from (unobservable) market clearing prices: Closing prices bounce arbitrarily between the closing bid and ask price. The bias is caused by the fact that a typical investor performing a rebalancing strategy is forced to buy at the ask price and to sell at the bid price. The bid-ask effect leads to an upward bias of closing price returns compared to the returns computed from actual market prices. In order to estimate the bias, we use *average* bid-ask spreads (BAS) for the individual companies. Using the effective spread for each week would be superior, but this cannot be done because the data is not available.

Is the bias expected to be important in our sample? As discussed in Section 2 of this paper, we impose a maximum (average) BAS of 20% as selection criterion for our liquid sample. Given the fact that 25% (31) of the vehicles in this sample have a BAS of 10% or more, we expect a considerable impact of the bid-ask effect. Our estimate is based on the following procedure: For the EW-RB index, weekly adjustments are required. We assume that investors buy at the ask price and sell at the bid price, and that the "true price" equals the arithmetic average. They thus lose half of the BAS in each transaction. This amount is subtracted from all EW-RB portfolio returns in every week throughout the sample period. The same procedure is performed for the EW-BH index after new listings take place. Naturally, the number of transactions is by far smaller than for the EW-RB index. The resulting, adjusted indices are displayed in **Exhibit 7**, and their statistical properties can be found in **Exhibit 8**.

Exhibit 7: The bid-ask spread bias in PTPE: The impact on a rebalanced and a buy-and hold strategy*





*The indices after bid-ask spread are calculated as follows: At the beginning an equal amount of capital is invested in the two indices. Remember that the EW-RB index is weekly rebalanced to equal weights while the EW-BH index is only rebalanced if a new vehicle is listed. We calculate the cost of every single transaction by assuming that an investor has to buy at the ask and sell at the bid price.
**Indices corrected for the bid-ask bias.

As is apparent from **Exhibit 8,** the BAS-effect for the EW-RB index is substantial. For the overall sample period, it amounts to a mean bias of 8.33%, in the down market from 2000-2003 the bias is even 10.95%. In contrast, the bias is only 0.39% for the EW-BH index.

Even after correcting for the bid-ask bias, the EW-RB index shows a clearly higher return compared to the EW-BH index in the period of 2000-2003. Through the diversification effect of the EW-RB index losses are limited in this period of declining stock markets although the underlying rebalancing strategy is costly as shown in this analysis.

Exhibit 8: The bid-ask spread in PTPE: The effect on rebalanced and buy-and-hold strategy

Time Period	EW-RB US$		
	Adjusted Mean	Standard Mean	Difference
1986 – 2003	***7.65%***	*15.99%*	*8.33%*
1986 – 2000	**19.45%**	27.21%	7.76%
2000 – 2003	**-46.34%**	-35.39%	10.95%

Time Period	EW-BH US$		
	Adjusted Mean	Standard Mean	Difference
1986 – 2003	***5.52%***	*5.91%*	*0.39%*
1986 - 2000	**22.04%**	22.26%	0.21%
2000 - 2003	**-70.12%**	-68.93%	1.20%

Annualized weekly returns are computed from 1st January 1986 - 19th March 2003, 1st January 1986 - 16th February 2000 and 16th February 2000 – 19th March 2003. They are based on 114 liquid companies.

4.3 Selection bias - ex ante selection bias and ex post survivorship bias

Two kinds of selection bias must be examined: The first is a potential selection bias caused by our selection rule. PTPE was a new and not well defined asset class in the mid-eighties; while our definition and selection rule (see Section 2) is adequate for the past few years, applying the same criteria *in retrospect* to the vehicles which were available 20 years ago, is to some extent arbitrary. However, the associated ex-post selection bias can at least be minimized if the selection rule is applied consistently over the whole sample period. We not only traced all new listings which fit our definition of a PTPE vehicle, but also assured that the selected vehicles still fit this definition throughout the entire time period.

The second type of selection bias is generally known as survivorship bias. The instruments surviving over a given sample period and thus representing the sample, need not to be a valid

representation of the entire set of instruments available to the investor over this time period. Excluding delisted vehicles generally leads to an upward bias in computed returns as has been demonstrated in various studies in the literature. The focus of most studies was on mutual fund performance, where the survivorship bias accounts for 0.1% to 1.5% annualized return bias, depending on the instruments (stocks, bonds), the time period, and the sample of funds.[20]

We noticed in Section 2 that from the overall sample of 287 firms, only 237 vehicles survived our observation period. 50 vehicles disappeared from the sample for the following reasons:

- Nine vehicles were acquired by other companies; in four cases they were acquired by another PTPE company which was already included in our sample.
- A group of 18 vehicles changed their businesses. Often these companies merged with one of their subsidiaries and decided to concentrate on the business of the latter.
- We found 15 vehicles that were delisted from the exchange or whose trading was suspended. In eight cases we could not exactly trace either the reasons for delisting or the exact date of disappearance from the stock exchange. Thus we excluded these eight vehicles from the analysis.

The remaining 42 vehicles were all delisted in the years 2000, 2001 and 2002. To be consistent with our analysis in this paper, we will only estimate a survivorship bias for those vehicles out of the 42 which satisfied our liquidity criteria.[21] Eight vehicles satisfy our liquidity constraints while the other 34 vehicles did not, especially in the time period right before they disappeared from the sample. Four out of these eight vehicles were delisted or trading was suspended, two vehicles were acquired, and the remaining two changed their businesses.

In order to quantify the survivorship bias caused by the eight vehicles, we adopt the "follow the money" procedure of Elton/Gruber/Blake (1996) based on raw returns.[22] This means that if a vehicle is delisted for whatever reason, the capital is equally allocated across all remaining vehicles. The survivorship bias is then measured by comparing the returns of a survivorship-free

[20] See e.g. Grinblatt/Titman (1989), Blake/Elton/Gruber (1993), Brown/Goetzmann (1995), Malkiel (1995), Elton/Gruber/Blake (1996), Carhart (1997), Carhart/Carpenter/Lynch/Musto (2000).
[21] An indication of the survivorship bias for the overall sample will be given in section 5.
[22] Elton/Gruber/Blake (1996) reported risk adjusted measures of survivorship bias in addition to measures based on raw returns.

index which includes the eight delisted vehicles (i.e. contains all 122 liquid vehicles) with our previous index which is just based on the 114 (liquid) surviving vehicles.

The results are displayed in **Exhibit 9** for three different time periods. The surprising result is that both the EW-RB and the EW-BH indices exhibit a "positive" survivorship bias over the entire sample period: including the non-surviving vehicles leads to higher returns (!). To our knowledge, no similar result has been reported in the performance literature. A possible explanation is that bad company performance explains only four of the eight delistings. In two cases, two high performing instruments were subsequently acquired, and two vehicles that changed their business also demonstrated a better performance than the surviving ones. This could explain the positive survivorship bias.

We conclude that the adjustment for survivorship bias is not as important as correcting for autocorrelation and the bid-ask spread. Yet the results could be quite different if we would have included all non-surviving, also the illiquid vehicles. This will be addressed in Section 5.

Exhibit 9: The survivorship bias in PTPE, liquid sample (114 vehicles)

Time Period	EW-RB US$		
	Adjusted Mean (n=122)	Standard Mean (n=114)	Difference
1986 - 2003	***16.35%***	*15.99%*	*0.36%*
1986 - 2000	**27.85%**	27.21%	0.64%
2000 - 2003	**-36.29%**	-35.39%	-0.90%

Time Period	EW-BH US$		
	Adjusted Mean (n=122)	Standard Mean (n=114)	Difference
1986 - 2003	***7.72%***	*5.91%*	*1.81%*
1986 - 2000	**22.44%**	22.26%	0.18%
2000 - 2003	**-59.68%**	-68.93%	9.25%

Annualized weekly returns are computed from 1st January 1986 - 19th March 2003, 1st January 1986 - 16th February 2000 and 16th February 2000 – 19th March 2003. They are based on 114 liquid companies.

5. Selecting a PTPE Benchmark Index

A major issue with respect to creating a PTPE benchmark index is still unresolved: the determination of the index universe.[23] We have somehow arbitrarily defined a set of liquidity constraints to get a sample of 114 "liquid" vehicles (see **Exhibit 1**). In this Section, we investigate the sensitivity of our empirical results for EW-RB PTPE indices with respect to different sample sizes: the full sample (EW-RB 287), the surviving sample (EW-RB 233), and a very small sample of firms represented in a "Major Market" index (EW-RB 16). A small and liquid index covering some ten to 20 instruments may be well suited for practical purposes, e.g. serve as the underlying for derivatives or structured products. The constraints imposed for this index are displayed in **Exhibit 10**, and the resulting index contains 16 vehicles.

A comparison of the mean returns of the various indices is presented in **Exhibit 11**. The average annual return of the EW-RB 233 index is 24.82% for the full sample time period 1986-2003, which is only marginally less than the average return of 25.29% computed for the same strategy applied to the full sample (the EW-RB 287 index). Hence, the positive survivorship bias documented earlier for the liquid sample is also present in the full sample.[24] Interestingly, the average return of the Major Market Index (EW-RB 16) is very close to the respective value of the EW-RB 114 (15.40% vs. 15.99%). This observation holds for the full sample period, as well as for the two subperiods (27.35% vs. 27.21% for 1986 to 2000; and –39.30% vs. –35.39% for 2000 to 2003). This demonstrates that the specification of our Major Market Index, although done very mechanically, provides a reasonable tracking of the liquid sample (EW-RB 114) - despite the correlation between the two indices may appear rather low (see below).

The standard deviations of the two indices are virtually identical (17.03% and 17.07%), but slightly lower than for the more liquid EW-RB 114 portfolio (19.34%), while the standard deviation of the Major Market Index is substantially larger (32.65%). This suggests that illiquidity accounts for a substantial part of the volatility of PTPE returns. Of course, a more

[23] See e.g. Cordero/Dubacher/Zimmermann (1988).
[24] We use the same procedure for estimation of survivorship bias as already presented in 4.3.

precise interpretation requires an adjustment for the biases discussed in Section 4, which are likely to be extremely important in the full sample.

Exhibit 10: Liquidity constraints for the Major Market Index (EW-RB 16)

The Major Market Index is based on the following liquidity criteria:

Number of vehicles	Liquidity constraints				
	Minimum of weekly observations	**Minimum average market capitalization**	**Minimum relative trading volume**	**Minimum continuity of trade**	**Maximum average bid-ask spread**
16	50	50	1%	30%	5%

Exhibit 11: Comparison of potential PTPE benchmark indices, equally weighted and rebalanced (EW-RB)

EW-RB US$		EW-RB 287		EW-RB 233		EW-RB 114		major market EW-RB 16	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD
	1986 - 2003	*25.29%*	*17.03%*	*24.82%*	*17.07%*	*15.99%*	*19.34%*	*15.40%*	*32.65%*
Time	1986 - 2000	34.30%	16.05%	32.83%	16.08%	27.21%	18.44%	27.35%	33.04%
Period	2000 - 2003	-14.83%	20.08%	-10.88%	20.30%	-35.39%	21.68%	-39.30%	29.72%
Number of Vehicles		287		233		114		16	

Annualized weekly returns are computed from 1st January 1986 - 19th March 2003, 1st January 1986 - 16th February 2000 and 16th February 2000 – 19th March 2003.

Exhibit 12 displays the correlation coefficients between the returns of all indices investigated so far. The correlation between EW-RB 233 and EW-RB 114 is 0.74, the correlation between EW-RB 114 and EW-RB 16 is 0.66. These figures are rather low – lower than the correlations between indices of similar size containing standard stocks. It should however be recognized that most stocks are listed in different countries, so that (strong) country effects may be present – beside thin trading or other effects lowering the estimates.

The performance characteristics of the Major Market Index (EW-RB 16) are shown in **Exhibit 13**. The calculated Sharpe ratios are 0.32 for the period 1986-2003, and 0.67 for 1986-2000.

Jensen's alpha for the time period of 1986 to 2003 is 9.19% with values of 16.54% and –22.66% for the subperiods. The beta ranges from 0.86 to 0.93. As expected and in line with Section 4, the degree of (significant) autocorrelations is by far lower for the Major Market Index than for the index constructed from the 114 liquid vehicles. This demonstrates that the high degree of autocorrelation in the larger indices can be indeed explained by thin trading.

Exhibit 12: Correlation coefficients between potential benchmark indices

	EW-RB 287	EW-RB 233	EW-RB 122	EW-RB 114	EW-RB 16	EW-BH 114	VW-BH 122
EW-RB 287	1.00	0.96	0.75	0.72	0.44	0.59	0.45
EW-RB 233		1.00	0.75	**0.74**	0.43	0.59	0.43
EW-RB 122			1.00	0.94	0.54	0.75	0.55
EW-RB 114				1.00	**0.66**	0.73	0.52
EW-RB 16					1.00	0.41	0.46
EW-BH 114						1.00	0.55
VW-BH 122							1.00

Exhibit 13: The Major Market Index (EW-RB 16)

EW-RB		US$ Mean	SD	Sharpe Ratio	Alpha	Beta	Autocorrelations AC1	AC2	AC3	AC4	AC5
	1986 - 2003	*15.40%*	*32.65%*	*0.32*	*9.19%*	*0.89*	*0.099***	*-0.010*	*0.069**	*0.065*	*0.054*
	1986 - 2000	27.35%	33.04%	0.67	16.54%	0.86	0.089*	-0.011	0.087*	0.085*	0.057
Time Period	2000 - 2003	-39.30%	29.72%	n.c.	-22.66%	0.93	0.137	0.060	-0.059	-0.195*	0.045

*/** denotes that the correlation is significant at the 5%/1% level.
Annualized weekly returns are computed from 1st January 1986 - 19th March 2003, 1st January 1986 - 16th February 2000 and 16th February 2000 – 19th March 2003.

Due to our high liquidity requirements applied to the Major Market Index (**Exhibit 10**), we should expect that the biases discussed in Section 4 are not as severe as for the other indices. In fact, as shown in **Exhibit 14**, the adjusted estimates do not substantially differ from the standard ones. For example, the unadjusted return of the Major Market Index from 1986-2003 is 15.40%,

while the bid-ask-spread-adjusted return is 12.07%, yielding a difference of only 3.33%. In the last column of Exhibit 14, the adjusted Sharpe Ratio including both the bid-ask bias and the volatility bias are reported. Due to the lower level of autocorrelation in the EW-RB 16 index, the implied adjustment of the volatility is relatively low. Even after adjusting for the two biases, the Major Market Index still yields a Sharpe ratio of 0.17 over 1986-2003, and 0.43 over 1986-2000.

Exhibit 14: Major Market Index (EW-RB16) – adjusted estimates

EW-RB	US$ Mean	SD	Sharpe Ratio	Bid-ask spread adjusted Mean	Volatility adjusted SD	Adjusted Sharpe Ratio*
Time Period 1986 - 2003	15.40%	32.65%	0.32	**12.07%**	**42.24%**	**0.17**

*The Sharpe ratio is based on the adjusted return and the adjusted volatility.
Annualized weekly returns are computed from 1st January 1986 - 19th March 2003.

As shown in this Section, the advantage of a rather small and liquid PTPE index is the fact that potential biases do not arise that strongly as for indices with a broader universe of vehicles such as the EW-RB 114. Yet concerning the choice of a valid benchmark index for the PTPE asset class as a whole, other criteria such as regional distribution and sector diversification would have to be taken into account. It is questionable whether this can be accomplished by a small index. Further research is required in this area.

6. Conclusion

In contrast to former studies estimating the performance of private equity by analyzing book values of the underlying investments, this paper presents a different approach by studying market prices of publicly traded private equity (PTPE) vehicles. We could identify a total of 287 PTPE vehicles which have been successively listed over the time period from 1986 to 2003. Our sample contains all listings on international exchanges, with a certain bias towards Europe (163) due to a high number of UK companies (113).

In order to get accurate measures of risk and return, several liquidity constraints are imposed. They have a strong impact on the size of the resulting sample. The major part of the empirical work conducted in this paper is based on a subsample including 114 liquid stocks[25].

We investigate the risk and return characteristics of two partially rebalanced indices (one equally-weighted, the other value-weighted) and a fully rebalanced (equally weighted) index. Not surprisingly, the performance of PTPE is high in the period from 1986 to 2000. The Sharpe ratios of 1.19 and 0.74 exceed the respective value of the MSCI (World) stock market (0.47). The results dramatically change if the subsequent years (up to March 2003) are included in the analysis. Compared with a Sharpe ratio of 0.09 for MSCI (World), only the fully rebalanced equally weighted strategy has a clearly superior performance (Sharpe ratio 0.57) – the two other strategies exhibit ratios close to zero. This demonstrates the importance of the portfolio or index style (in terms of weighting and rebalancing) in studying the performance of PTPE.

Because the liquidity of most PTPE vehicles is small compared to traditional stock market investments, several performance biases should be taken into account. Our investigation reveals that standard volatility estimates are strongly downward biased due to the artificial autocorrelations in PTPE returns. Taking into account the adjusted risk estimate, the Sharpe ratio for the fully rebalanced EW-RB 114 index decreases from 0.57 to 0.33 and is still above the MSCI (World) index. We moreover investigate the so called “bid-ask bias” in our rebalancing

[25] Eight non-surviving vehicles were studied separately in section 4 reducing the original liquid sample of 122 vehicles to 114.

strategies. In case of the EW-RB 114 index, the mean annual return correction is 8.33% over the entire sample period, which is dramatic. Thus, essentially the entire return premium of the rebalanced strategy disappears by taking into account the bid-ask spread in portfolio adjustments. We finally report estimates of the survivorship bias in our liquid sample by including the (eight) non-surviving vehicles in our analysis. Surprisingly and in contrast to the literature, we find a small positive bias in essentially all periods.

In Section 5 we compare the statistical properties of several potential PTPE benchmark indices. The return estimates range from 15.40% for a small Major Market Index (EW-RB 16) up to 25.29% for the index covering the entire sample (EW-RB 287). The small index exhibits quite favorable investment characteristics – small biases, close returns to the overall liquid sample (EW-RB114), high volatility, which makes it a potential candidate as underlying for structured PTPE products or derivatives.

Appendix

Appendix 1: Country classification of Publicly Traded Private Equity Companies

Region/Country	number of companies	percentage	market value [Mio. US$]	percentage	mean market value [Mio. US$]	median market value [Mio. US$]
America	**82**	**28.57%**	**13'645.84**	**24.90%**	**99.43**	**9.23**
US	61	21.25%	13'462.95	24.57%	131.84	14.58
Canada	21	7.32%	182.89	0.33%	8.37	1.80
Asia	**31**	**10.80%**	**19'741.37**	**36.02%**	**636.88**	**18.95**
Australia	8	2.79%	86.23	0.16%	10.89	8.56
Hongkong	1	0.35%	31.50	0.06%	31.53	31.53
Japan	3	1.05%	19'029.52	34.72%	6343.17	4147.15
South Korea	18	6.27%	586.14	1.07%	32.63	20.60
Sri Lanka	1	0.35%	7.98	0.01%	7.98	7.98
Europe	**163**	**56.79%**	**20'951.15**	**38.23%**	**118.11**	**27.45**
Austria	2	0.70%	80.74	0.15%	40.37	40.37
Belgium	2	0.70%	1'324.87	2.42%	662.78	662.78
Denmark	1	0.35%	156.35	0.29%	156.35	156.35
Finland	1	0.35%	8.97	0.02%	8.97	8.97
France	6	2.09%	464.79	0.85%	41.58	28.21
Italy	2	0.70%	729.56	1.33%	99.60	99.60
Germany	16	5.57%	1'879.12	3.43%	94.42	70.68
Netherlands	3	1.05%	1'054.38	1.92%	351.51	428.17
Spain	1	0.35%	102.92	0.19%	12.92	12.92
Sweden	9	3.14%	1'235.90	2.26%	154.69	31.33
Switzerland	7	2.44%	1'551.00	2.83%	134.27	68.68
UK	113	39.37%	12'362.55	22.56%	108.95	24.37
Rest of the world	**11**	**3.83%**	**466.86**	**0.85%**	**35.97**	**19.26**
Israel	3	1.05%	291.66	0.53%	73.22	76.39
South Africa	6	2.09%	133.11	0.24%	22.31	13.29
Turkey	2	0.70%	42.09	0.08%	21.05	21.05
Total	**287**	**100.00%**	**54'805.22**	**100.00%**	**190.96**	**22.45**

Datasource: Primark Datastream

Appendix 2: Impact of liquidity constraints

Average market capitalization [Mio. US$]	overall sample	after liquidity constraints	difference
< 10	75	13	62
10 to 30	94	38	56
30 to 50	37	16	21
50 to 100	26	14	12
100 to 250	30	25	5
250 to 500	11	8	3
500 to 1000	8	3	5
1000 to 5000	3	2	1
> 5000	3	3	0
total	**287**	**122**	

[Mio. US$]	Sum	1 - 10	11 - 50	51 - 122
Market cap.	37'421	29'710	6'047	1'670
Percentage	100.00%	79.39%	16.15%	4.46%

Average bid-ask spread	overall sample	after liquidity constraints	difference
smaller than 5%	82	66	16 (20%)
5% - 10%	36	25	11 (31%)
10% - 15%	25	18	7 (28%)
15% - 20%	20	13	7 (35%)
20% - 25%	15	4	11 (73%)
25% - 50%	23	4	19 (83%)
greater than 50%	86	54	32 (37%)
total	**287**	**184**	

Datasource: Primark Datastream

Appendix 3: Spearman correlations between number of observations, relative turnover, bid-ask spread, continuity of trade and market capitalization

Correlations

		Observations	Relative trading volume	Average bid-ask spread	Continuity of trade	Average market value
Observations	correlation coefficient	1	0.09	-0.38**	0.25**	0.27**
	t-value	-	0.1641	0.000	0.000	0.000
Relative trading volume	correlation coefficient		1	-0.15*	0.14	0.15*
	t-value		-	0.046	0.336	0.0182
Bid-ask spread	correlation coefficient			1	-0.23**	-0.48**
	t-value			-	0.000	0.000
Continuity of trade	correlation coefficient				1	0.32**
	t-value				-	0.000
Average market value	correlation coefficient					1
	t-value					-

*/** denotes that the correlation is significant at the 5%/1% level.

Appendix 4: Impact of the specification of liquidity constraints on sample size

Number of companies	Liquidity constraint				
	Minimum of weekly observations	Minimum average market capitalization	Minimum relative trading volume	Minimum Continuity of trade	Maximum average bid ask spread
122	30	2	0.10%	15%	20%
102	30	3	0.10%	40%	20%
81	30	5	0.10%	40%	10%
101	20	3	0.10%	40%	20%
95	100	3	0.10%	40%	20%
66	200	3	0.10%	40%	20%
102	30	3	0.10%	40%	20%
98	30	5	0.10%	40%	20%
92	30	10	0.10%	40%	20%
98	30	5	0.10%	40%	20%
71	30	5	0.30%	40%	20%
61	30	5	0.60%	40%	20%
117	30	5	0.10%	15%	20%
103	30	5	0.10%	30%	20%
98	30	5	0.10%	40%	20%
85	30	5	0.10%	30%	10%
96	30	5	0.10%	30%	15%
106	30	5	0.10%	30%	25%

Appendix 5: The autocorrelation structure of different PTPE indices

Autocorrelations

01/1986 - 03/2003

	VW-BH 114	EW-RB 114	EW-BH 114	MSCI (World)
AC 1	0.114**	0.151**	0.058	-0.022
AC 2	-0.012	0.159**	0.150**	0.060
AC 3	0.053	0.115**	0.007	0.049
AC 4	0.037	0.099**	0.072*	-0.061
AC 5	0.075*	0.077*	0.098**	-0.014
Q 5	20.588	69.337	36.518	9.234
p-value	0.001	0.000	0.000	0.100

*/** denotes that the correlation is significant at the 5%/1% level.

Appendix 6: Market model regressions

EW-RB 114		Standard Estimates			Adjusted Estimates		
		Beta estimates	Estimated R^2	Alpha	Dimson Beta estimates	Estimated R^2	Dimson Alpha
	1986 - 2003	*0.60*	*0.22*	*10.18%*	*0.99*	*0.25*	*10.43%*
	1986 - 2000	0.58	0.18	18.21%			
time period	2000 - 2003	0.60	0.30	-25.83%			

EW-BH 114		Standard Estimates			Adjusted Estimates		
		Beta estimates	Estimated R^2	Alpha	Dimson Beta estimates	Estimated R^2	Dimson Alpha
	1986 - 2003	*0.74*	*0.17*	*-0.09%*	*1.09*	*0.18*	*-0.05%*
	1986 – 2000	0.72	0.18	12.37%			
time period	2000 - 2003	0.72	0.13	-56.74%			

VW-BH 114		Standard Estimates		
		Beta estimates	Estimated R^2	Alpha
	1986 - 2003	*1.20*	*0.17*	*-1.20%*
	1986 – 2000	1.08	0.13	7.16%
time period	2000 - 2003	1.41	0.29	-31.18%

*/** denotes that the correlation is significant at the 5%/1% level.

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